

Vibrational Spectroscopy 2

Polyatomic Molecules

Recommended Reading:

Banwell and McCash Chapter 3

Atkins Chapter 16

Vibrational Modes

Diatomic molecules have 1 bond \Rightarrow 1 vibrational mode = bond stretch.

In polyatomic molecules bond **lengths** and **angles** can change

\Rightarrow many different modes.

How many vibrational modes does a molecule have?

Consider a molecule with N atoms.

1. Need 3 co-ordinates (x,y,z) to specify position of each atom

\Rightarrow $3N$ co-ordinates to specify shape of molecule

\Rightarrow **$3N$ degrees of freedom**

2. Need 3 co-ordinates to specify translational motion (centre of mass)

\Rightarrow **$3N - 3$ degrees of freedom left**

3. Need 3 co-ordinates (angles) to specify orientation of molecule

\Rightarrow **$3N - 6$ vibrational modes (non-linear)**

4. If molecule is linear only need 2 angles to specify orientation

\Rightarrow **$3N - 5$ vibrational modes (linear)**

Bending and Stretching Modes

If molecule has N atoms, then it will have $N-1$ bonds and each bond can undergo a stretching vibration.

⇒ $N-1$ stretching modes

this then leaves

⇒ $(3N - 6) - (N - 1) = 2N - 5$ bending modes (non-linear molecule)

⇒ $(3N - 5) - (N - 1) = 2N - 4$ bending modes (linear molecule)

A **Normal Mode** is a vibration that can be excited without exciting any of the other modes ⇒ no energy transfer between normal modes.

Any vibration of the molecule can be expressed as a **linear sum** of the Normal Modes of the molecule

Each normal mode can be considered as an independent oscillator with its own values of k , μ , and $\tilde{\nu}_0$

Each vibrational mode has its own set of energy levels

Each vibrational degree of motion corresponds to a separate type of coordinated overall motion of all of the atoms in the molecule, which is called a **normal mode** of vibration. For a general polyatomic molecule the total vibrational energy is conventionally written as

$$\varepsilon_{\text{vib}}(n_1, n_2, n_3, \dots) = \sum_i \tilde{\nu}_i \left(n_i + \frac{1}{2} \right) + \sum_i \sum_{j>i} \chi_{i,j} \left(n_i + \frac{1}{2} \right) \left(n_j + \frac{1}{2} \right) + \dots$$

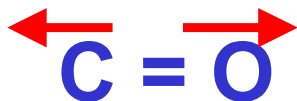
where n_i is the vibrational quantum number for mode i , ν_i and $\chi_{i,j}$ are the harmonic and leading anharmonic vibrational constants.

So in order to calculate the actual vibrational spacing associated with any particular mode it is necessary to know both the constants ν_i and $\chi_{i,i}$ associated with that mode but also all of the mixed anharmonicity constants $\chi_{i,j}$ for $j \neq i$.

We now look at some specific simple examples:

Types of Vibration

In diatomic molecules the only vibrational mode is a simple stretching mode.



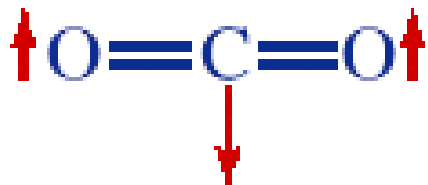
More unusual vibrational motions exist for the polyatomic case



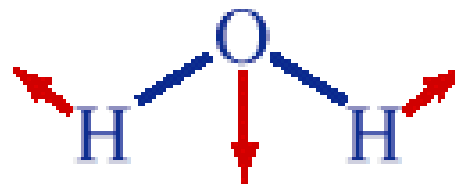
Symmetric stretch (breathing) mode



Antisymmetric stretch



Degenerate bending
modes



symmetric
deformation

Web site for animations:

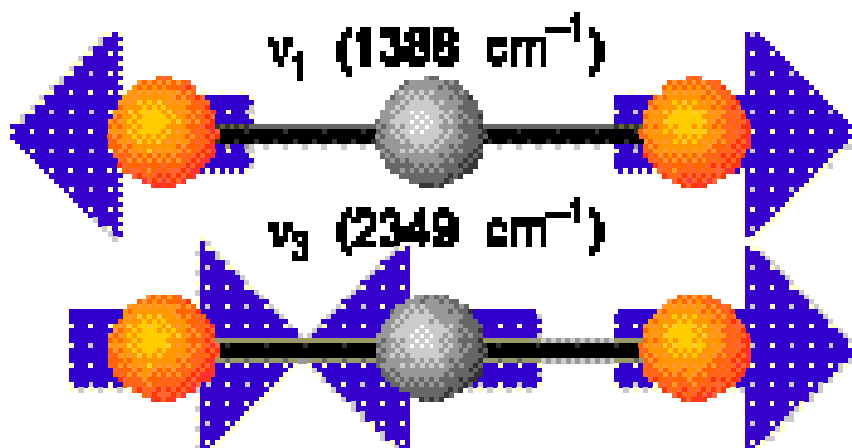
<http://www.ems.psu.edu/~bannon/moledyn.html>

Example: Vibrational Modes of CO₂

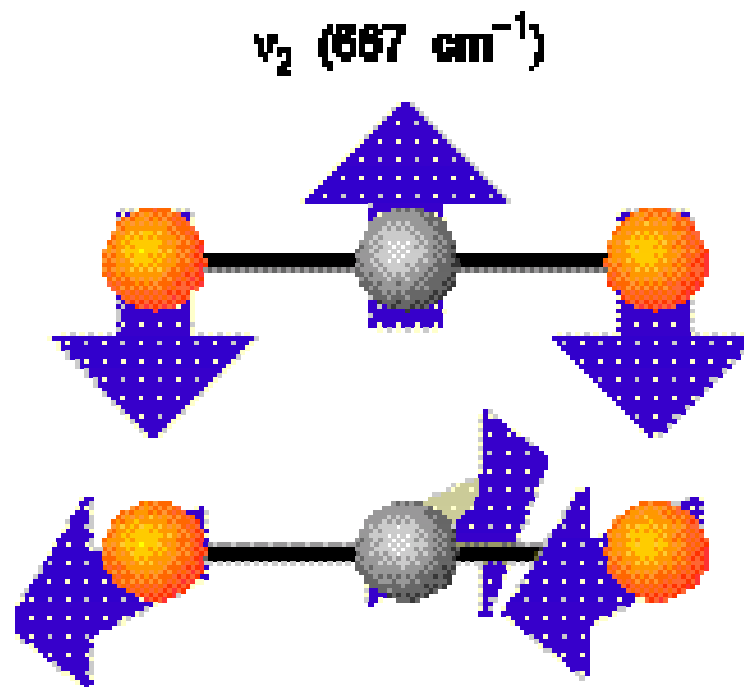
Three atoms, linear molecule: $N = 3 \Rightarrow 3N - 5 = 4$ vibrational modes

$N - 1$ Stretching Modes: $\Rightarrow 2$

$2N - 4$ Bending modes: $\Rightarrow 2$



Note that wavenumbers are in IR region of EM spectrum.



Bending modes are **degenerate** so they occur at the same energy (wavenumber).

How many lines would we see in the vibrational spectrum of CO₂?

Hint: Don't forget selection rules!

IR Spectrum of CO₂

Only see TWO vibrational bands.

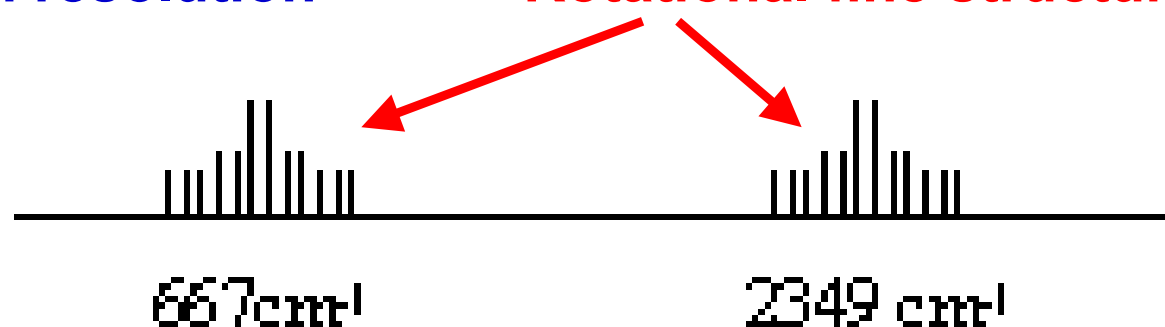
The symmetric stretch mode does not have a changing dipole moment. Not IR active.

In Solution: Low resolution, see two absorption bands



In Gas Phase: High resolution

Rotational fine structure



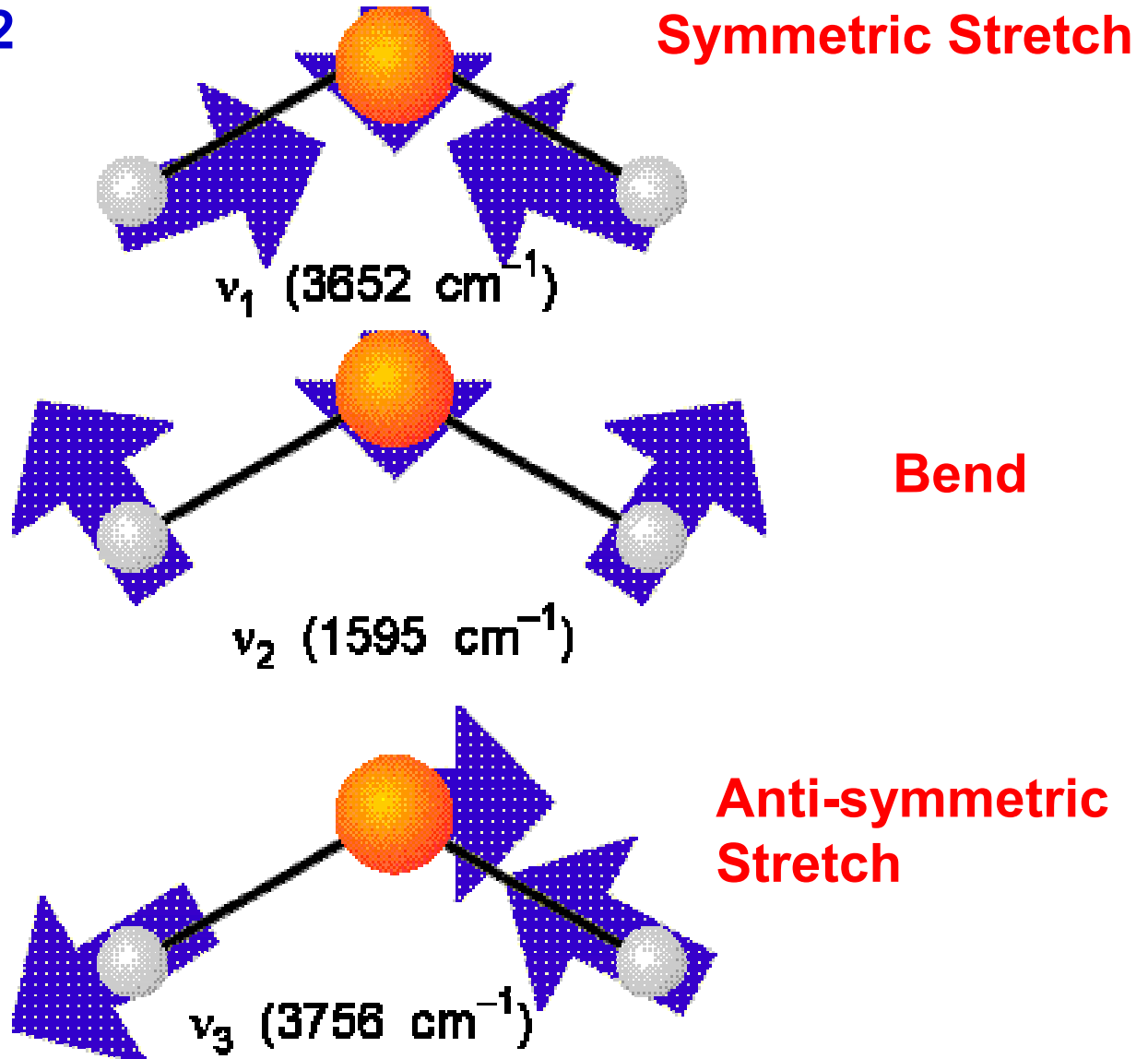
See notes later on vibrational-rotational spectra

Example: Vibrational Modes of Water H₂O

Three atoms, nonlinear molecule: $N = 3 \Rightarrow 3N - 6 = 3$ vibrational modes

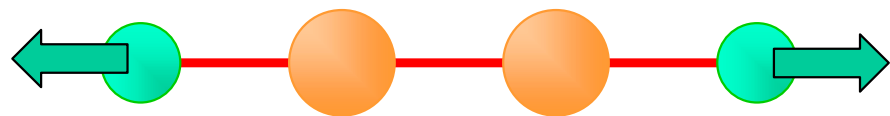
$N - 1$ Stretching Modes: $\Rightarrow 2$

$2N - 5$ Bending modes: $\Rightarrow 1$

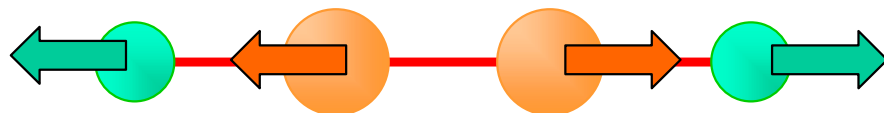


Vibrational Modes of Acetylene C₂H₂

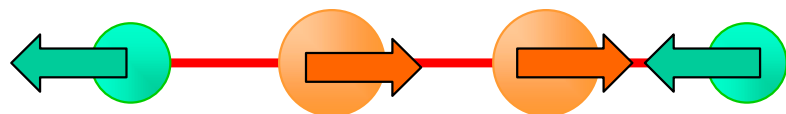
Linear molecule: $N = 4 \Rightarrow 3N - 5 = 7$ modes: 3 stretching and 4 bending



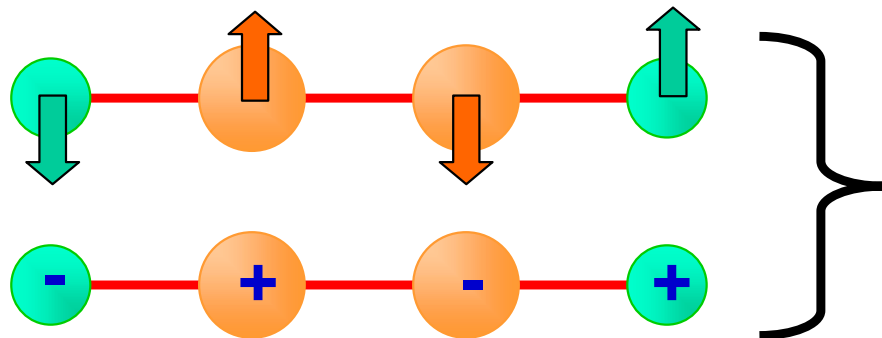
ν_1 , C-H symmetric stretch, 3347 cm⁻¹



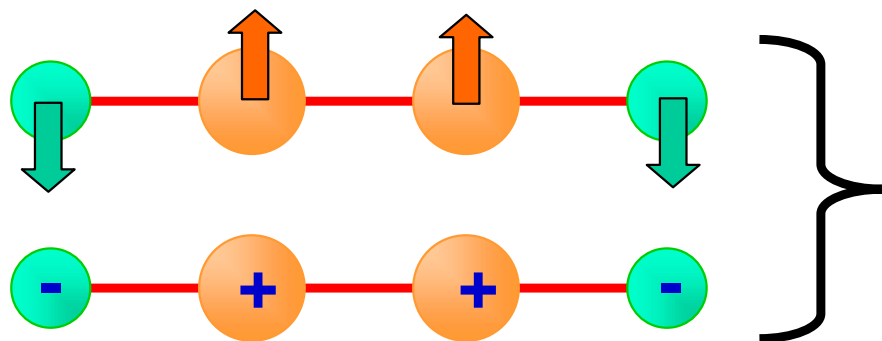
ν_2 , C=C stretch, 1974 cm⁻¹.



ν_3 , C-H antisymmetric stretch
3287 cm⁻¹

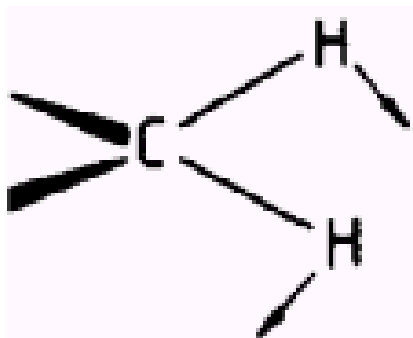


ν_4 , trans bend, 612 cm⁻¹
doubly degenerate

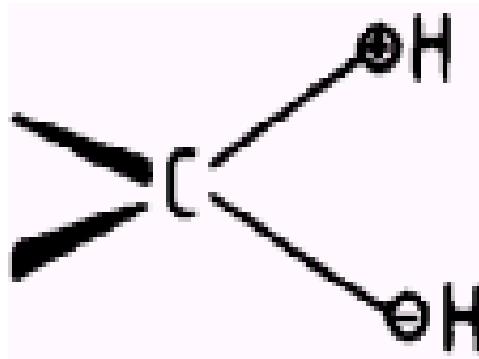


ν_5 , cis bend, 729 cm⁻¹
doubly degenerate

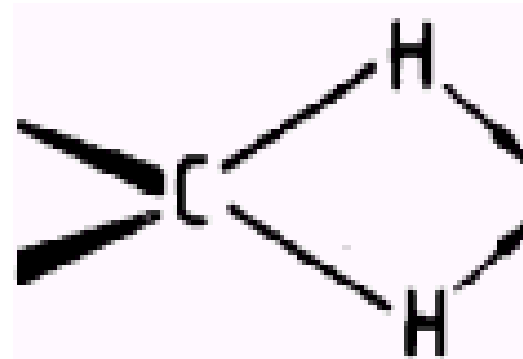
Many Types of Vibration



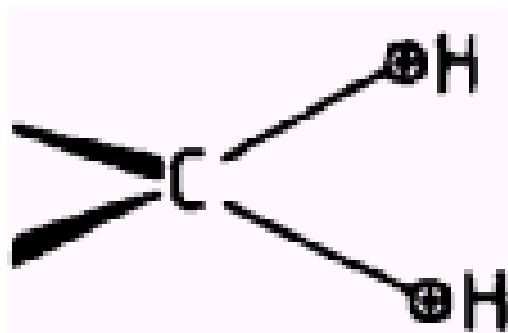
-CH₂ Rocking



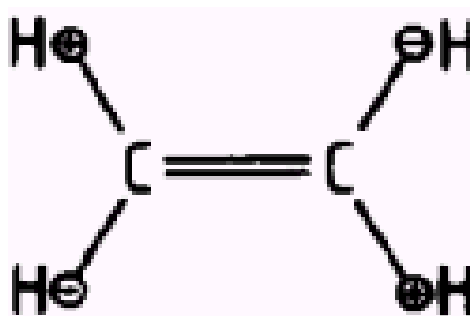
-CH₂ Twisting



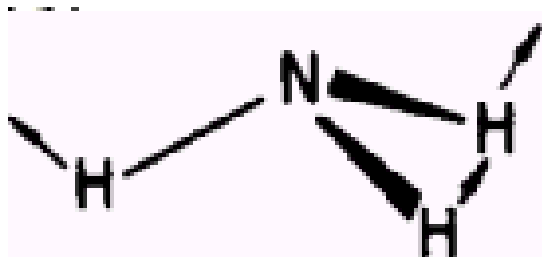
-CH₂ Scissoring



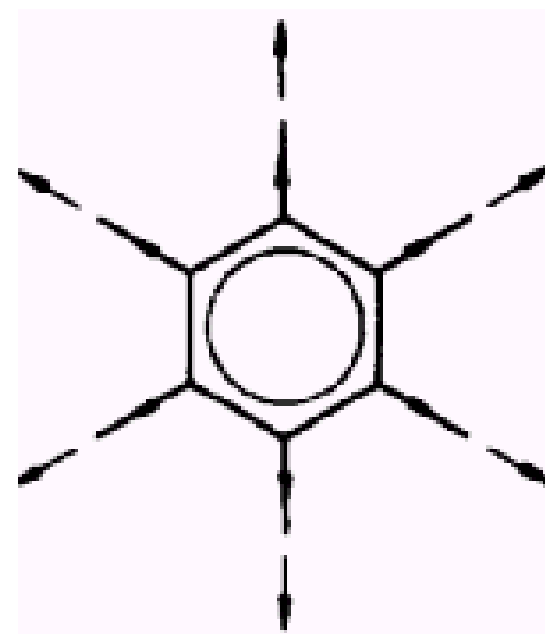
-CH₂ Wagging



**Torsional
Vibrationa**



**Inversion
(umbrella)
vibrationa**



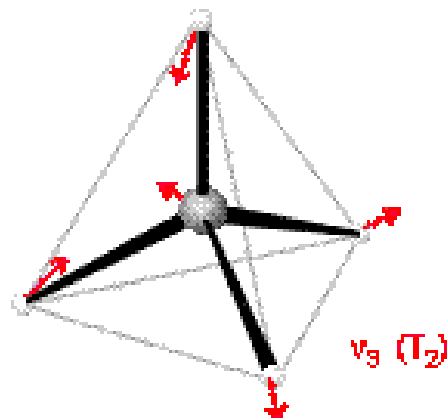
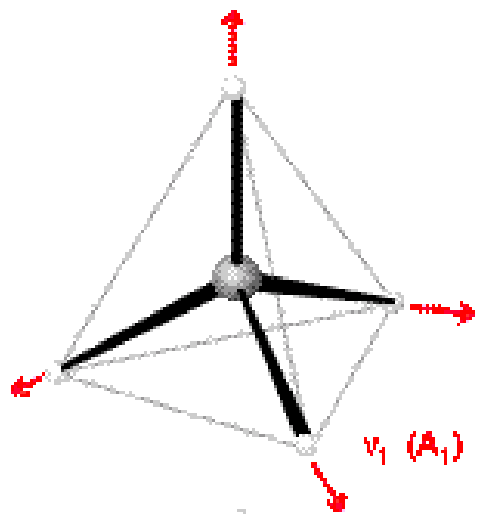
Breathing Mode

Example: Vibrational Modes of CH₄

Five atoms, non-linear molecule: $N = 5 \Rightarrow 3N - 6 = 9$ vibrational modes

$N - 1$ Stretching Modes: $\Rightarrow 4$

$2N - 5$ Bending modes: $\Rightarrow 5$

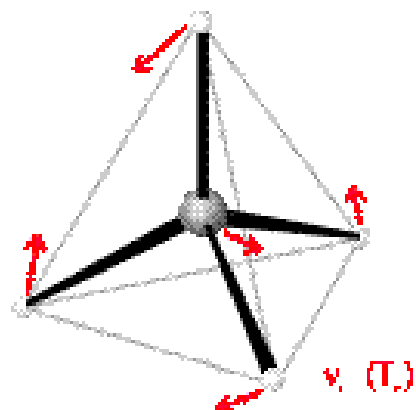
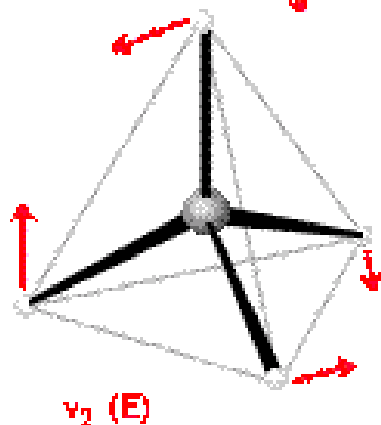


Some modes are degenerate so only 4 different modes observed.

Not all modes are infra red active - ν_3 and ν_4 are observed in infra red spectrum.

How do we know this?

GROUP THEORY - a full course by itself



Larger Molecules

Recall that

$$\tilde{\nu} = \frac{1}{2\pi c} \sqrt{\frac{k}{\mu}}$$

So each mode will have its own value of **k** and **μ**

3N-6 vibrational modes, so for a large molecule (N > 4) the IR spectrum can be very complex

Vibrational modes can be localised in parts of the molecule.

In some modes ALL of the atoms undergo displacements.

In other modes only small groups of atoms are vibrating

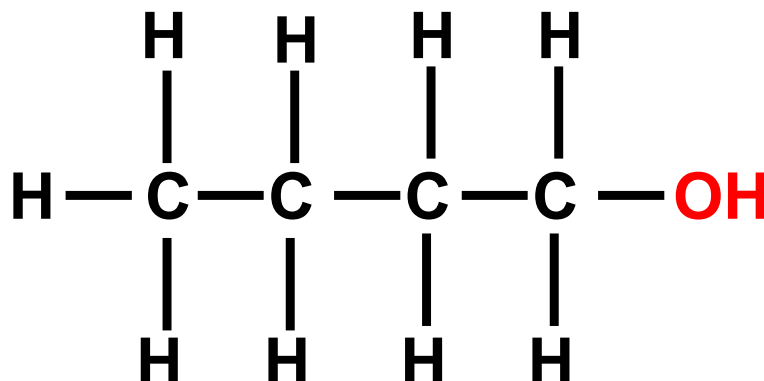
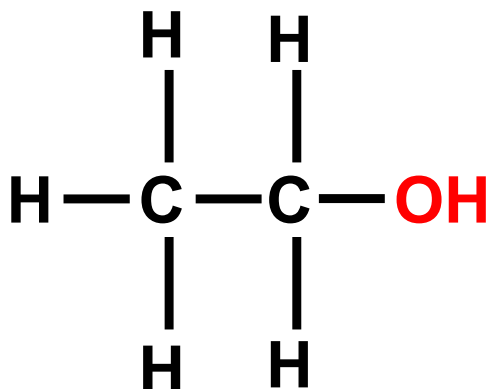
⇒ Two classes

1) **SKELETAL Vibrations**: Arise from strong coupling between stretching or bending motions of atoms in straight chains, branched chains or in a ring. Many atoms involved, all undergoing approximately the same displacement.

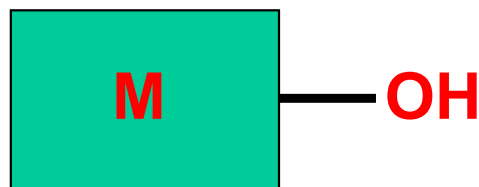
2) **CHARACTERISTIC GROUP Vibrations**: Almost independent of the structure of the molecule as a whole and just depend on the structure of the group undergoing vibration

Group Vibrations

Consider the following molecules (alcohols) $\text{C}_2\text{H}_5\text{OH}$, $\text{C}_4\text{H}_9\text{OH}$...



As far as the OH group is concerned we can represent both these molecules as



Where M is the 'large' mass of the molecule. The vibrational frequency of the OH group will then be given by

$$\omega_0 = \sqrt{\frac{k}{\mu}}$$

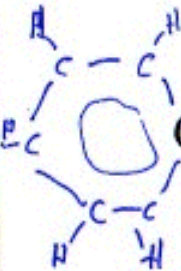
with

$$\mu = \frac{M \cdot m_{\text{OH}}}{M + m_{\text{OH}}} \approx m_{\text{OH}}$$

so to a first approximation, the vibrational frequency is independent of the overall size of the molecule and only depends on the mass of the vibrating group (OH in this case)

Characteristic Group Vibrations

IR absorption frequencies are characteristic of certain functional groups on the molecule.

Chemical Group	Approximate Frequency (cm^{-1})	Chemical Group	Approximate Frequency (cm^{-1})
 -OH	3600 <i>O-H stretch</i>	SH	2580
-NH ₂	3400	C≡N	2250
≡C-H	3300 <i>C-H stretch for ≡C</i>	C≡C	2220
C-H (phenyl)	3060 <i>C-H on benzene ring</i>	C=O	1750-1600
=CH ₂	3030 <i>C-H stretch for =C</i>	C=C	1650
-CH ₃ } <i>C-H stretch</i> {	2970 (asym. stretch)	C=N	1600
-CH ₃ } <i>C-H stretch</i> {	2870 (sym. stretch)	C-C, C-N, C-O	1200-1000
-CH ₃ } <i>C-H bend</i> {	1460 (asym. deform.)	C=S	1100
-CH ₃ } <i>C-H bend</i> {	1375 (sym. deform.)	C-F	1050
=CH ₂ - } <i>C-H stretch</i> {	2930 (asym. stretch)	C-Cl	725
=CH ₂ - } <i>C-H stretch</i> {	2860 (sym. stretch)	C-Br	650
=CH ₂ - } <i>H-H bend</i> {	1470 (deformation)	C-I	550

See Banwell and McCash Table 3.4

Trends in Group Frequencies

Recall that

$$\tilde{\nu} = \frac{1}{2\pi c} \sqrt{\frac{k}{\mu}}$$

Increasing the mass of the atom undergoing vibration within the group will tend to decrease the frequency (wavenumber). e.g



Increasing the strength of the bond, and hence the value of k , increases the frequency. e.g



wavenumbers are in the ratio

$$2220 : 1650 : 1200$$

$$1.85 : 1.38 : 1$$

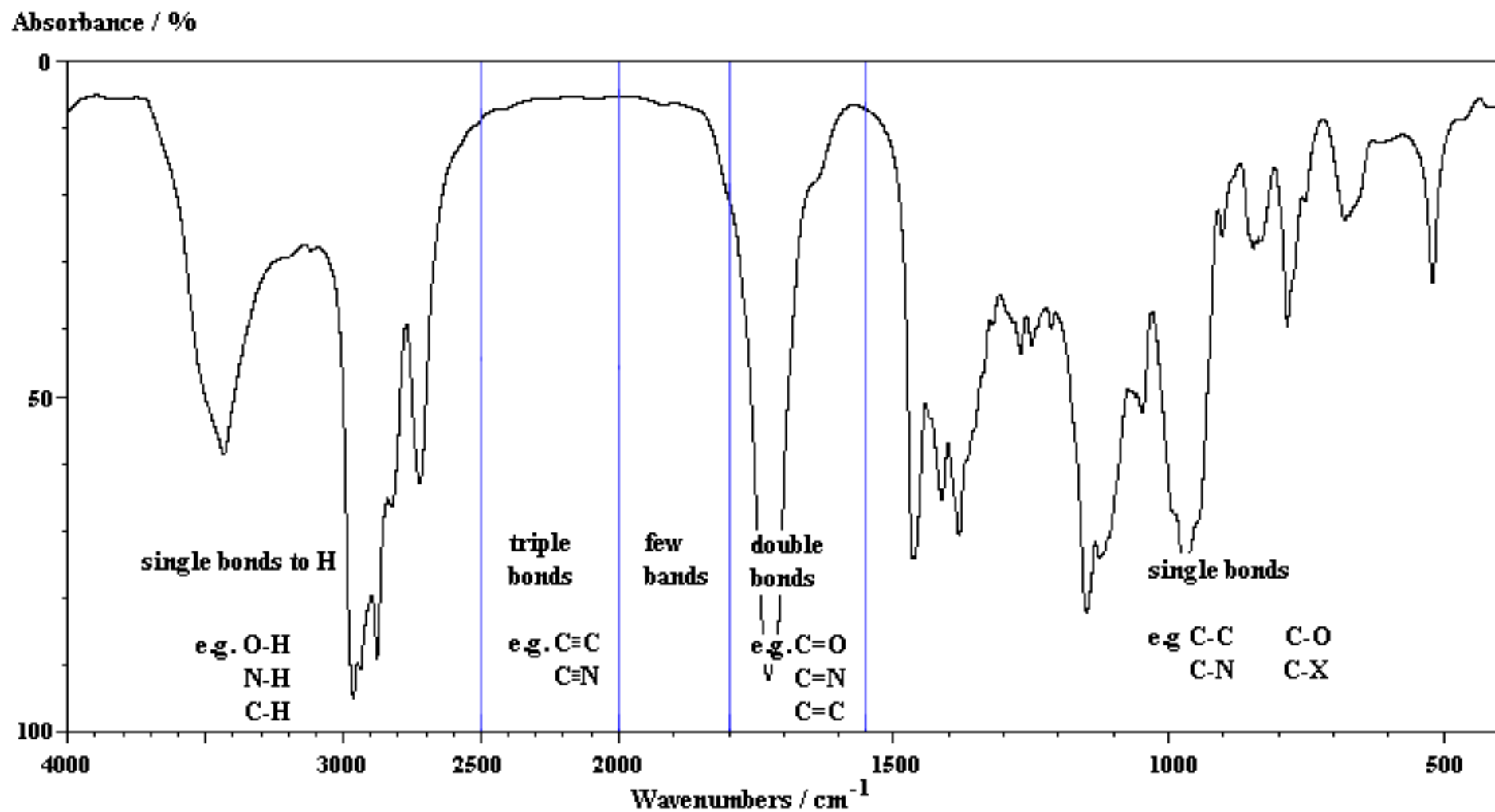
$$\sqrt{3} : \sqrt{2} : 1$$

as expected since $\tilde{\nu} \propto \sqrt{k}$

Intensities of the bands in the IR spectrum depends on the magnitude of the change in the dipole moment. In general the more polar the bond the larger the dipole moment and the stronger the band, e.g.

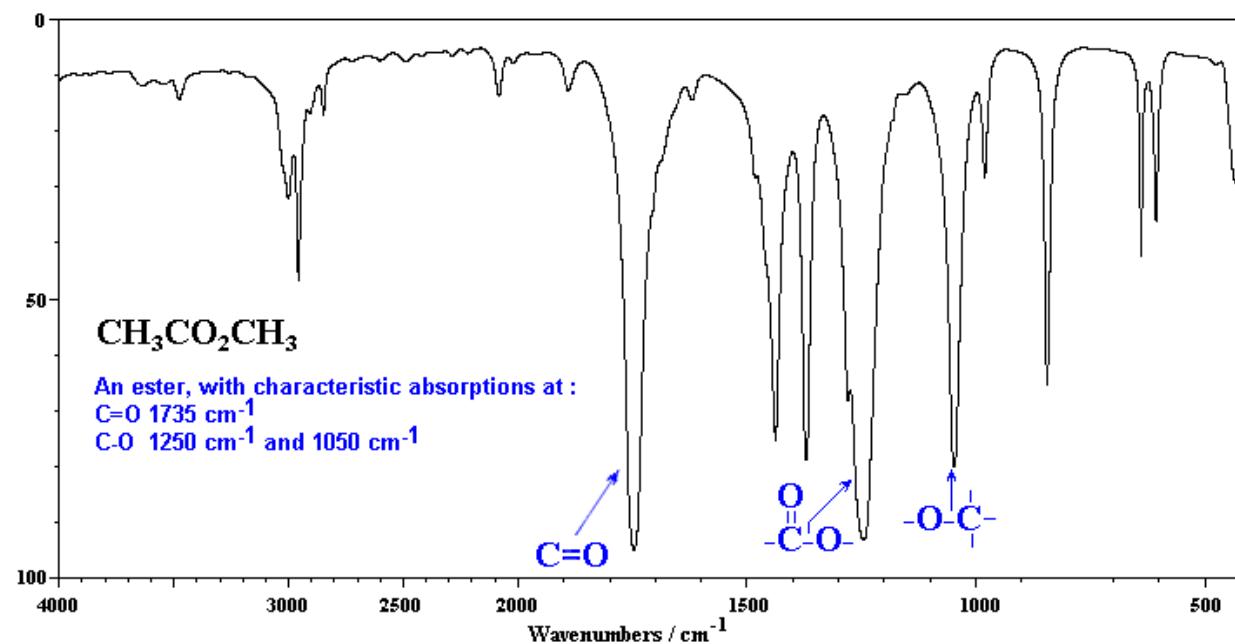


Examples

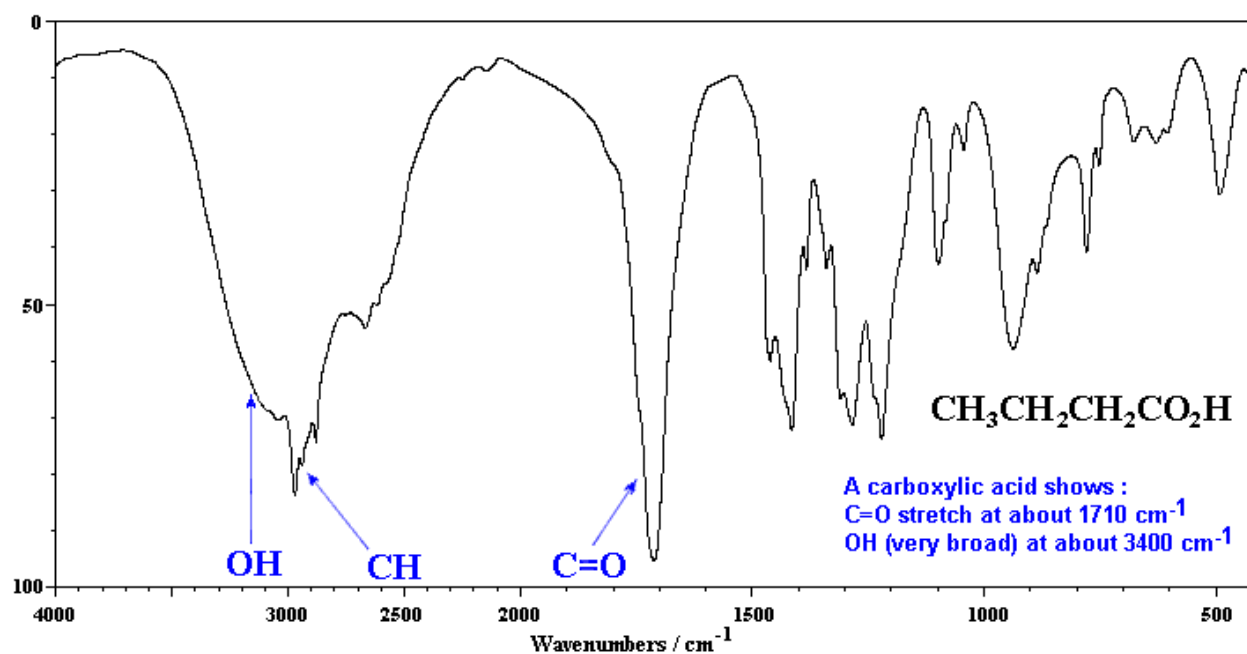


Examples

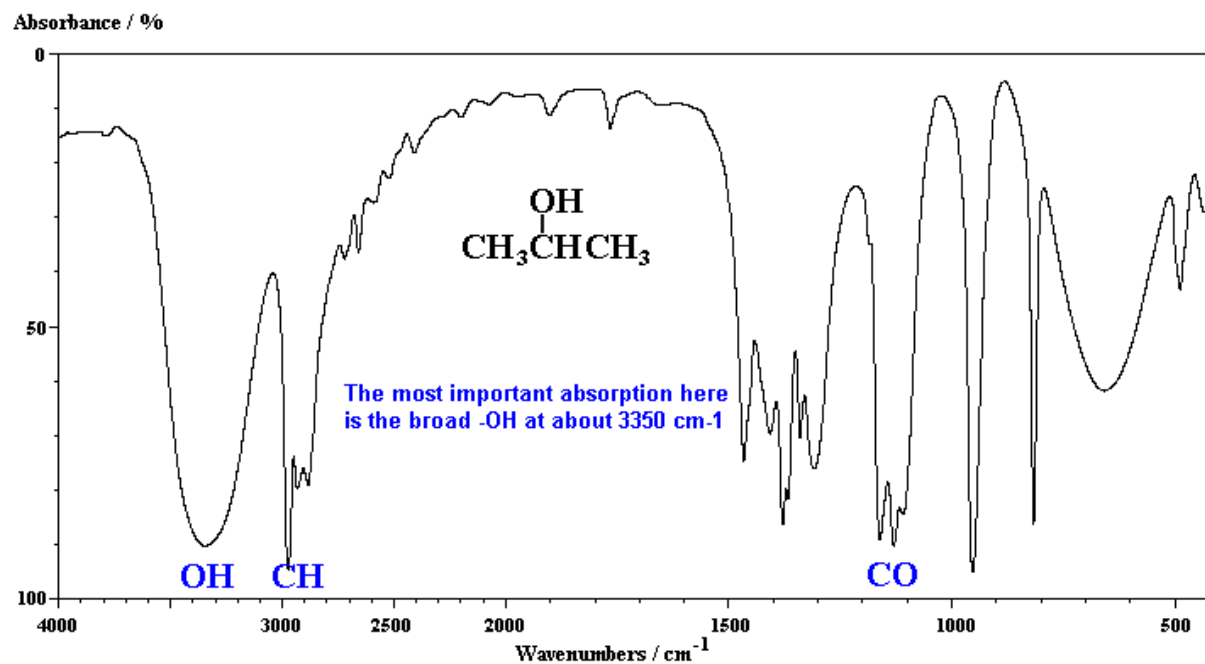
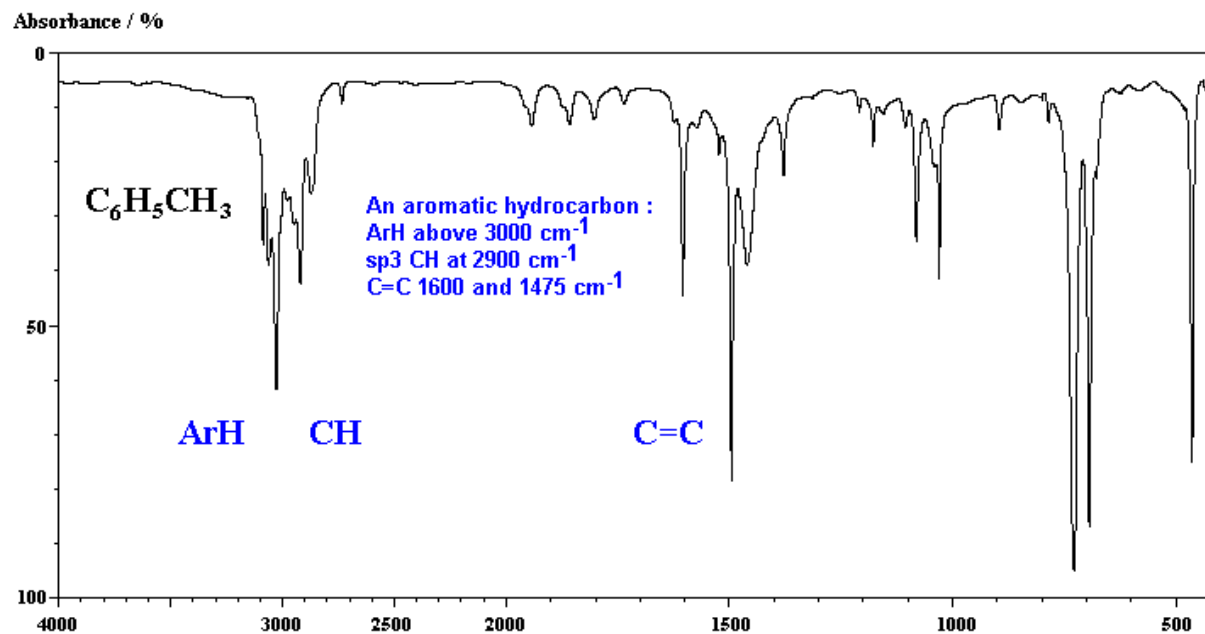
Absorbance / %



Absorbance / %

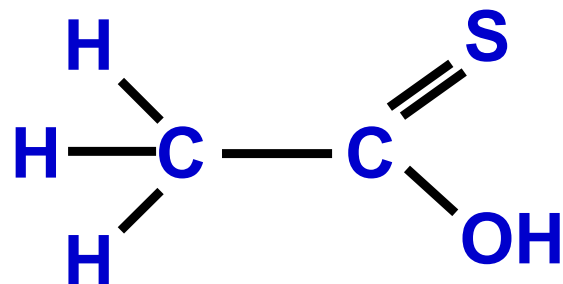


Examples

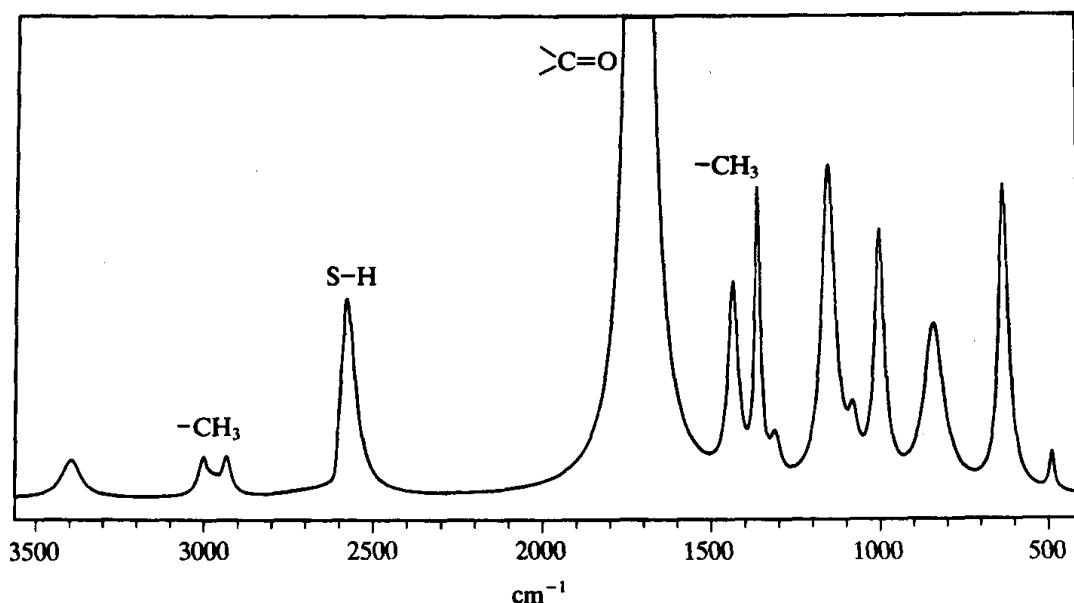
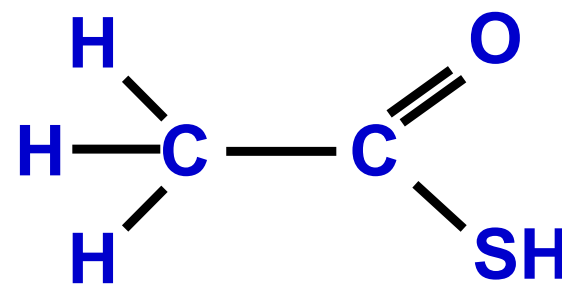


Example: Thioacetic Acid C_2H_4OS

Which structure?



or

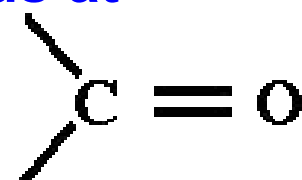


IR spectrum shows sharp absorption bands at

$1730 \text{ cm}^{-1} \Rightarrow$

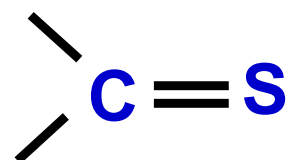
and

$2600 \text{ cm}^{-1} \Rightarrow$



$-\text{SH}$ group

No absorption near $1100 \text{ cm}^{-1} \Rightarrow$ no

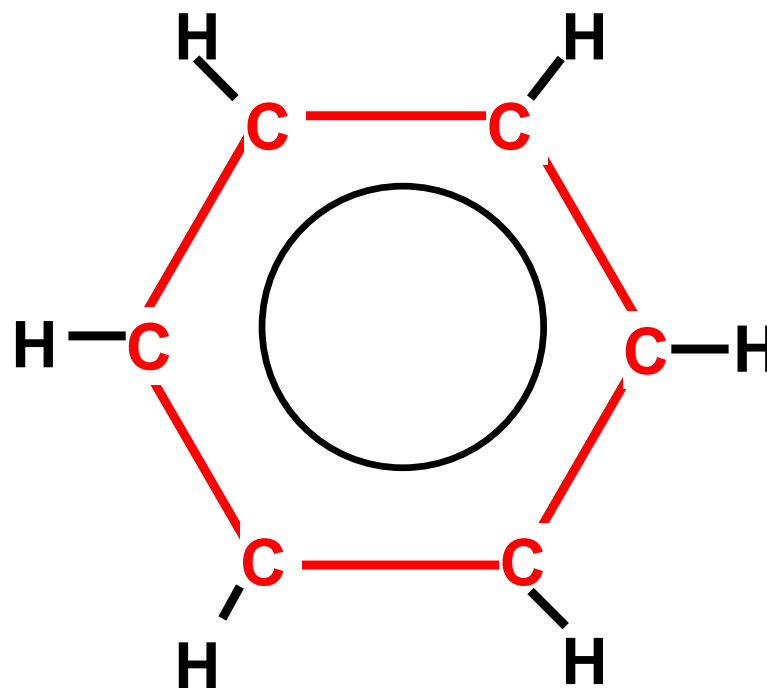
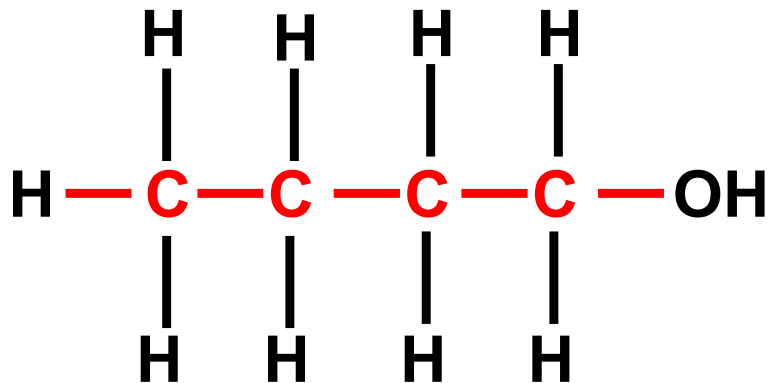


group

See Banwell and McCash figure 3.19(c)

Skeletal Vibrations

Arise from branched or linear chain structures. These are vibrations of the whole chain or along a side arm to a chain or ring structure. Because many atoms are involved in the vibration, the mass is high and therefore the vibrational frequency (and wavenumber) is low



Usually in the range **1400 - 700 cm^{-1}** for organic molecules